Chapter 10: Congruency and Similarity

Content Matrix

Sections	Student's Book	Workbook
	Chapter Opener, p. 1 Learning Outcomes, p. 1 Recall, p. 2	
10.1 Congruent Figures	Pause and Discover pp. 3 Key Results p. 3 **Worked Example 1** and **Pause and Try 1** pp. 4 – 5 Pause and Discover pp. 5 – 6 Key Results p.7 **Worked Example 2** and **Pause and Try 2** pp. 7 – 8 **Worked Example 3** and **Pause and Try 3** pp. 8 – 9 Pause and Discover p. 10 Key Results p.10 **Worked Example 4** and **Pause and Try 4** p. 11 **Worked Example 5** and **Pause and Try 5** p. 12 Exercise 10.1 pp. 13 – 16	Further Exercise 10.1 pp. 1 – 4
10.2 Ⓢ Congruency Tests	Ⓢ Pause and Discover pp. 17 – 18 Ⓢ Key Results p. 19 Ⓢ **Worked Example 6** and **Pause and Try 6** p. 20 Ⓢ **Worked Example 7** and **Pause and Try 7** p. 20 Ⓢ **Worked Example 8** and **Pause and Try 8** p. 21 Ⓢ **Worked Example 9** and **Pause and Try 9** p. 22 Ⓢ **Worked Example 10** and **Pause and Try 10** p. 23 Ⓢ **Worked Example 11** and **Pause and Try 11** p. 23 Ⓢ Exercise 10.2 pp. 24 – 25	Ⓢ Further Exercise 10.2 pp. 5 – 7
	Review p.26 Review Questions pp. 27 – 29	Chapter Opener Revisit, p. 8 Fun with Maths!, p. 8 Chapters 8–10 Revision Exercise, pp. 9 – 10

(Only selected lesson plans for Section 10.1 Congruent Figures and 10.2 Congruency Tests are included in this submission.)

LESSON 1
Warm-up

Chapter Opener (p. 1)
Begin the lesson by discussing with students the idea of 'identical' and 'similar' in maths. They are related to the geometrical terms 'congruent' and 'similar' respectively. These two geometrical ideas are commonly used in real-life. For example, architects and urban planners use these ideas to bring together ideas to plan and build new places or improve existing places to fulfil certain design considerations.

Invite students to briefly share about how the land can be divided based on the two proposals shown in the opener. Ask them to share some of their ideas behind their thinking.

Briefly mention that the use of identical and similar shapes is related to the ideas of congruency and similarity that they will be learning in this chapter

Note: The Chapter Opener will be revisited in the Workbook under Chapter Opener Revisit.

Learning Objectives
State the learning objectives that students will learn in the chapter:
- Use and interpret the geometrical terms similarity and congruence.
- Calculate lengths of similar figures.
- Recognise congruent shapes.
- Calculate lengths of similar figures.
- Use the relationships between areas of similar triangles, with corresponding results for similar figures and extension to volumes and surface areas of similar solids.
- Use the basic congruence criteria for triangles (SSS, ASA, SAS, RHS).

Recall (p.2)
Ask students to attempt the diagnostic questions.
[Answers are on MCEduHub]

Main Lesson Content

10.1 Congruent Figures (pp. 2 – 16)

Introduce the commonly-used vocabulary 'congruent' by getting students to recall two figures that are identical.

Get students to consider the sizes (or sides) and shapes (or angles) of the two identical or congruent figures.
Explain that two figures that have the same size (i.e. equal corresponding sides) and the same shape (i.e. equal corresponding angles) are congruent.

Pause and Discover (p. 3)

Draw students' attention to the objectives of the activity.
Distribute to each student a copy of TR01 that contains 4 pentagons. Give the class a hint that two of the figures are identical. Encourage the class to discuss the meaning of 'identical' in this case. Invite some students to explain what they would do to determine which two figures are identical.
Instruct students to cut out each figure carefully. Assign sufficient time for students to carry out the activity and elicit answers.

Ask: How can we show that two figures are identical? Why are they or why are they not identical? What do we call the figures that are identical?
[Answer: We can overlap the figures to check if they overlap exactly. If they do, they are identical. They are identical because all they have the same size and shape. We call figures that are identical congruent figures.]

Emphasise to students that despite the different orientations, Figures *A* and *D* are identical. Highlight and write 'Figures *A* and *D* are congruent.' on the board. Explain that it means that the figures are identical.

Pause and Think
Invite students to discuss the meaning of the term 'identical'.
Ask: What does 'identical' mean mathematically?
[Answer: It means that two figures have the same size (i.e. equal corresponding sides) and the same shape (i.e. equal corresponding angles), that is, they are congruent.]

Key Results (p. 3)

Have students fill in the blanks to summarize the idea of congruence.

- Congruent figures have the same size (i.e. equal corresponding sides) and same shape (i.e. equal corresponding angles). They are identical but they may have different orientations.

Wrap-up

Review the lesson by summarising the key points. By the end of the lesson, students should understand:

- how to determine if a given set of figures are congruent

LESSONS 2 AND 3

Warm-up

Briefly recap what students have learnt about congruence in the earlier lesson. Explain that they will need to apply the Key Results from the previous lesson as they go through the worked examples in the lesson.

Main Lesson Content

Go through **Worked Example 1**. After going through the worked example, have students attempt **Pause and Try 1**.

Worked Example 1 and **Pause and Try 1 (pp. 4 – 5)**

- Student will learn to determine if two figures are congruent.
- Photocopy the page and cut out each of the figures.
- **(a)** *Ask: How would you find out if the figures are the congruent? Is the method reliable or effective? Why?*
  [Answers: I can overlap cut-outs of the figures. If they have the same size and shape, they will overlap exactly. This shows that the two figures are congruent. I can also compare the corresponding sides and angles of the figures. If they are equal, the figures are congruent. It is reliable to use the cut-outs or compare the figures' corresponding sides and angles to prove congruency. But it is more effective to do comparisons because it is a quicker method.]
- Demonstrate to the class that the two figures fit together exactly. Highlight that congruent figures have the same size and shape.
- **(b)** *Ask: What would you need to determine to conclude that the given pair of figures are congruent?*
  [Answer: I need to determine if their corresponding sides and angles are equal. The given pair of figures have equal sides, but they have equal corresponding angles. So they are not congruent.]
- Overlap the cut-outs of the two figures. Highlight that even though the length of the sides are equal, the size of the angles are not equal. So the two figures are not congruent.
- **(c)** *Ask: The two figures are rectangles. How do you determine if they are congruent?*
  [Answer: Since the figures have the same shape, I need to determine if they have equal length and breadth. If they do, they are congruent.]
- Overlap the cut-outs of the two figures. Highlight that even though both are rectangles with the equal angles, the lengths of the rectangles are not equal. So the two figures are not congruent.

  TWM1 (TWM.04 Convincing)
  Learners will show they are convincing when they justify why two figures are congruent or not by comparing their size (or corresponding sides) and shape (or corresponding angles).

Wrap-up

Review the lesson by summarising the key points. By the end of the lesson, students should understand:

- naming of two congruent triangles

Assign **Exercise 10.1** for students to do as practice for consolidation of concepts and skills.
For further practice, assign **Further Exercise 10.1** in the Workbook to students.

LESSONS 5 AND 6

Warm-up

10.2 Congruency Tests (pp. 17–25)

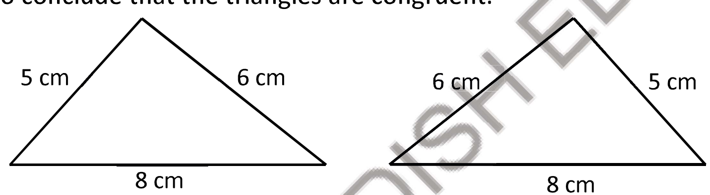
Briefly recap what students have learnt about congruent figures (i.e. all corresponding sides are equal and all corresponding angles are equal) and how congruent figures can be obtained through reflection, rotation and translation.
Discuss with students the 6 measures (3 angles and 3 sides) associated with a triangle. Then invite students to share the different measures needed to construct triangles.

Main Lesson Content

Pause and Discover (pp. 17 – 18)

1(a) Invite students to discuss and explore whether they can construct two different triangles with sides 8 cm and 5 cm using a ruler and compass or graphing software. Lead them to conclude that two different triangles can be constructed if only two sides are known.

(b) Invite students to discuss and explore whether they can construct two different triangles with sides 8 cm, 5 cm and 6 cm using ruler and compass or a graphing software. Guide them to conclude that the triangles are congruent.



Direct students' attention to the **Interesting Fact** on the page. Ask students to construct a few triangles to show the Triangle Inequality theorem works.

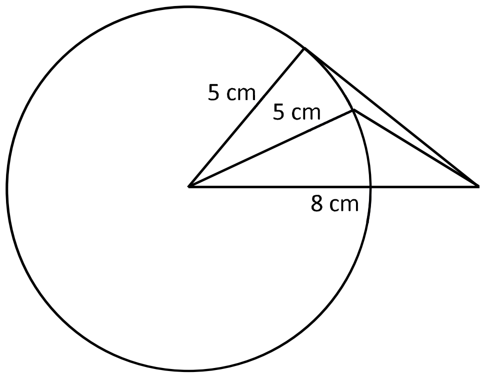
TWM3 (TWM.01 Specialising)
Students will show they are **specialising** when they construct a few triangles to show that the triangles satisfy the Triangle Inequality theorem.

Ask: How can you show that the given triangles are congruent? How can you obtain one triangle from another using a geometrical transformation?
[Possible Answers: I can check that the triangles have the same size and shape. I can translate/reflect/rotate one triangle to obtain the other triangle.]

> **Pause and Think**
> Explain to students why the conclusion will still be the same regardless of the lengths that they choose.

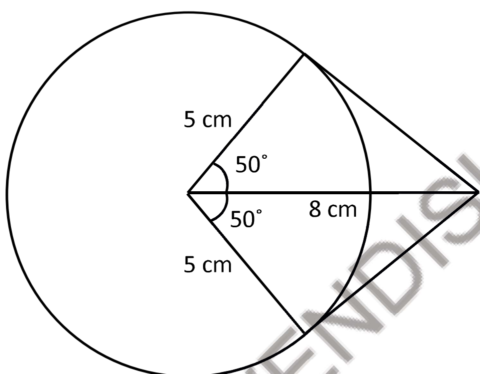
(c) Conclude that knowing 3 sides when constructing a triangle uniquely determines a triangle and this is called the Side-Side-Side (SSS) Congruency Test.

2(a) Ⓢ Invite students to construct a circle with radius 5 cm. Then use the radius as one of sides of each triangle to construct two different triangles with sides 8 cm and 5 cm.



Invite students to suggest another condition (other than the third side) such that a unique triangle can be determined.

Ask: What condition needs to be included to make the triangles congruent?
Possible Answer: An angle between the two given sides, or the included angle.]

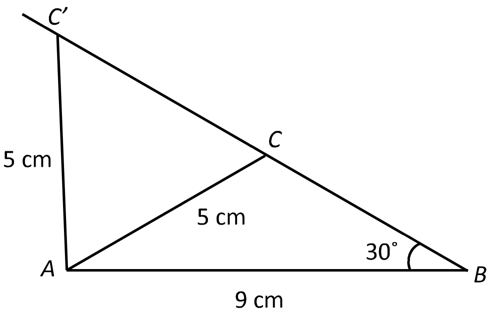


Pause and Think
Use the diagram to help students see why the included condition ensures the congruency of the two triangles. Guide them to see that when the angle between 5 cm and 8 cm is a fixed angle, both triangles must be congruent.

Conclude that knowing 2 sides and the included angle when constructing a triangle uniquely determines a triangle and this is called the Side-Angle-Side (SAS) Congruency Test.

(b) Invite students to construct the figure shown in the Student's Book. Ask them how many different triangles can be constructed. For students who need support, invite them to construct a circle with radius 5 cm with centre at *A*.

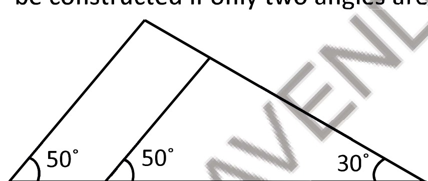
Ask: How many different locations can you draw Point C? How many different triangles can you draw?
[Answer: 2; 2]



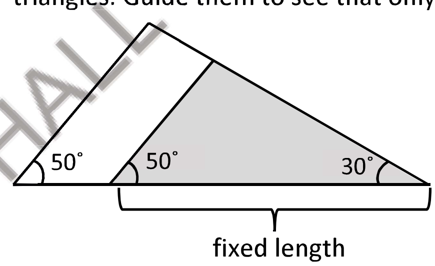
Pause and Think
Ask students to share the similarities and differences in the conditions of the SAS and SSA. Guide them to conclude that the difference is in the given angle.

Conclude that the SSA may not be a congruence test if the given angle is not the included angle.

3(a) Invite students to construct a triangle that has angles 50° and 30°. Encourage them to compare their work with their classmates' and discuss their observations. Invite volunteers with different triangles and show them to the class. Highlight that many different triangles can be constructed if only two angles are given.



(b) Provide students with a dimension for *AB* and ask them to try to construct different triangles. Guide them to see that only a unique triangle can be constructed.

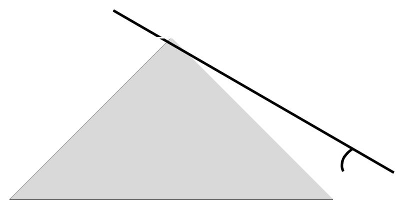


(c) Provide students with the dimension of *AC* instead and ask them to try to construct different triangles. Guide them to see that only a unique triangle can be constructed.

l students to conclude that Angle-Side-Angle (ASA) is a congruency test. Ask them to ain why AAS can be considered congruency test. Encourage them to give examples to /ince their classmates.

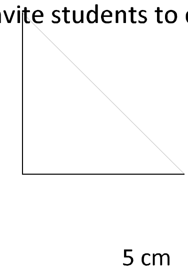
TWM4 (TWM.03 Conjecturing)
Learners will show they are **conjecturing** when they suggest ideas that form a congruency test.

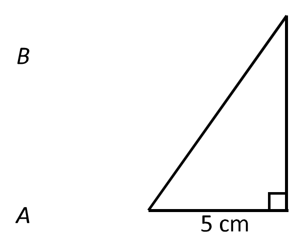
TWM5 (TWM.04 Convincing)
Learners will show they are **convincing** when they use the triangles they have constructed to justify why ASA is a congruency test and AAS is not a congruency test since when two angles are known, the third angle can always be determined.



Conclude that knowing 2 angles and a given side when drawing a triangle uniquely determines a triangle and that this is the Angle-Angle-Side (AAS) or Angle-Side-Angle (ASA) congruency test.

4 Invite students to construct as many $\triangle ABC$ with $AB = 5$ cm, $AC = 7$ cm and $\angle ABC = 90^\circ$.





Ask them to conclude that these triangles are con

Ask: How do you know that these triangles are congruent?
[Possible Answer: They can be mapped onto each other by reflection and/or rotation.]

Conclude that in a right-angled triangle, knowing one side and the hypotenuse when drawing a triangle uniquely determines a triangle and that this is the Right-Angle-Hypotenuse-Side (RHS) congruency test.
Highlight to students that the RHS congruency test is only used when the hypotenuse is equal. Otherwise, other congruency tests have to be used.

Note
ight to students that while SSA is not a congruence test, the RHS Congruency Test is a al case of the SSA.

References:

1	Congruent Triangles	• Hang, K.H & Wang, H (2017). *Solving Problems In Geometry: Insights And Strategies For Mathematical Olympiad And Competitions* (pp. 1–34). World Scientific
2	Exploring Side-Side-Angle Criterion	• Cirillo, M., Todd, R., & Obrycki, l J. (2015). *Exploring side-side-angle triangle congruence criterion.*

Key Results (p. 19)

Have students fill in the blanks to summarise the four congruency tests.

1. (i) If the 3 sides of a triangle are equal to the 3 corresponding sides of another triangle, then the two triangles are congruent.
   (ii) This means that all the corresponding sides and angles are equal.
   (iii) This is called the SSS Congruency Test.

2. (i) If the 2 sides and the included angle of a triangle are equal to the 2 corresponding sides and the corresponding included angle of another triangle, then the two triangles are congruent.
   (ii) This is called the SAS Congruency Test.

3. (i) If the 2 angles and 1 side of a triangle are equal to the 2 corresponding angles and the corresponding side of another triangle, then the two triangles are congruent.
   (ii) This is called the ASA Congruency Test.

4. (i) If the hypotenuse and 1 side of a right-angled triangle are equal to the hypotenuse and 1 side of another right-angled triangle, then the two triangles are congruent.
   (ii) This is called the RHS Congruency Test.

Wrap-up

Review the lesson by summarising the key points. By the end of the lesson, students should Understand how to:

- Use the basic congruence criteria for triangles (SSS, ASA, SAS, RHS) to test for congruency

LESSONS 7 AND 8

Warm-up

Briefly recap what students have learnt about the four different congruency tests – SSS, SAS, AAS and RHS in the earlier lesson. Explain that they will need to apply the Key Results from the previous lesson as they go through the worked examples in the lesson.

Main Lesson Content

Go through **Worked Examples 6 and 7**. At relevant junctures after going through each worked example, have students attempt **Pause and Try 6 and 7**.

Worked Example 6 and **Pause and Try 6 (p. 20)**

- Students will learn to prove that two triangles are congruent using the SSS Congruency Test.
- Emphasise the importance of presenting the proof in a systematic manner. Present the proofs in such a way that the measurement on the left side of the '=' are the sides and angles of one triangle and the measurements on the right side of the '=' include those from another triangle. Highlight that each statement needs to be justified with a reason for equality.
- *Ask: What are the conditions given in the question?*
  [Answer: $AC = EF = 3.8$ cm, $BC = DF = 2.6$ cm, $AB = ED$]

Worked Example 7 and **Pause and Try 7 (p. 20)**

- Students will learn to prove that two triangles are congruent using the SAS Congruency Test.
- Guide students to see that since there are 2 pairs of equal corresponding sides, the SAS congruency test should be used.
- *Ask: What are the conditions given in the question? What is the congruency test that you should use? Which included angle did you use?*
  [Answer: $HL = LK$, $GL = JL$. I should use the *SAS* Congruency Test with $\angle GLH = \angle JLK$.]

Wrap-up

Review the lesson by summarising the key points. By the end of the lesson, students should be able to:

- use the basic congruence criteria for triangles (SSS, ASA, SAS, RHS) to prove that two triangles are congruent

Assign **Exercise 10.2** for students to do as practice for consolidation of concepts and skills. For further practice, assign **Further Exercise 10.2** in the Workbook to students.

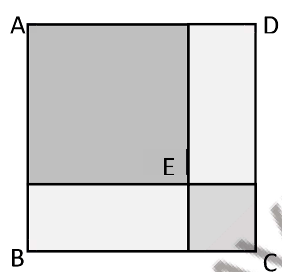
Lesson Plans for Section 10.3 Similar Figures, Section 10.4 Areas of Similar Figures and Section 10.5 Volumes of Similar Figures are not included in this submission.

Direct students to the list of learning outcomes under **Review** and have them put a tick in the boxes for all the learning outcomes that they have achieved.

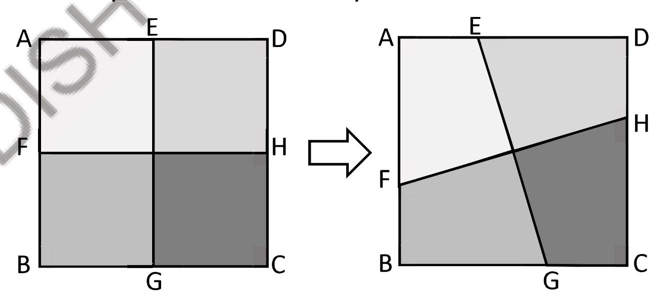
Assign **Review Questions** as practice for students to consolidate and reinforce the concepts and skills covered in the chapter.

Go through the **Chapter Opener Revisit** in the Workbook. Students should be able to apply what they have learned in this chapter to solve the problem presented in the Chapter Opener.

- Discuss the pros and cons of both methods.
- *Ask: In Proposal 1, are all squares similar? If the ratio of the areas of the squares is 1 : 4, what is the ratio of the sides of the squares? How will you decide which facility to build on which area?*
  [Possible Answers: Squares of different sizes are similar. The ratio of the areas is equal to the square of the ratio of their sides. I divided the piece of land into 4 parts for each proposal. The largest area can be used for the hotel building or the shopping arcade. A theme park will require more space than a swimming pool. So, the size of the part of the land for the theme park must be larger than that of the swimming pool also.]
- *Ask: In Proposal 2, simplify the proposal to dividing the land to 4 identical squares first. Think of how you can modify the squares into identical quadrilaterals by using suitable geometrical transformations. Is this proposal realistic?*
  [Possible Answers: I can use rotation about the centre of the land to generate four congruent quadrilaterals. But it is not practical to have the areas of the four facilities to be identical, i.e., having the same size, since different facilities call for different sizes. My method meets the criteria of the proposals, but I need to check that the methods are feasible in real-life.]
- After discussion, use a graphing software to explain and show some possible solutions.



Proposal 1



Proposal 2

To conclude the chapter, go through **Fun with Maths!** In the Workbook. This task encourages students to explore and discover when a line that is parallel to the base of a triangle is drawn, the lines of the two sides of the resulting two similar triangles will have an interesting ratio.

- Allow students ample time to explore either using a graphing software or on graph paper.
- Next, invite students to share their observations.
- Ask students to write down a proof to show that their observation is true for any triangle.

Since $\triangle ABC$ is similar to $\triangle ADE$, let $\frac{AB}{AD} = \frac{AC}{AE} = r > 1$.

$\frac{AB}{AD} = \frac{AD + DB}{AD} = r$

This means that $AD + DB = rAD$.

$AD + DB = rAD$

$AD(r-1) = DB$

$\frac{AD}{DB} = \frac{1}{r-1}$

Similarly, $\frac{AC}{AE} = \frac{AE + EC}{AE} = r$

This means that $AE + EC = rAE$.

$AE + EC = rAE$

$AE(r-1) = EC$

$\frac{AE}{EC} = \frac{1}{r-1}$

Thus, $\frac{AD}{DB} = \frac{AE}{EC}$.

Teacher's Resources

TR01 (Pentagons)