CHAPTER 12

Binomial Theorem

Further Exercise 12.1 Binomial Expansion

1 **(a)** Write down the coefficients of the terms in the expansion $(x + y)^7$.

(b) Hence, expand $(x + y)^7$.

2 Using the expansion of $(x + y)^7$ in Question 1 above, expand $(a - b)^7$.

3 **(a)** Expand $(1 + 3d)^6$. What do you observe about the powers of d in the expansion?

(b) Find, in ascending powers of d, the first three terms in the expansion of $(2 - d)(1 + 3d)^6$.

(c) Find the coefficient of d^4 in the expansion of $(1 + d)(1 + 3d)^6$.

4 By finding the first three terms in the expansion of $(1 + x)^6$ in ascending powers of x, find the value of $(1.02)^6$ correct to 2 decimal places.

5 The row of coefficients in the expansion of $(x + y)^7$ is 1, 7, 21, 35, 35, 21, 7, 1.
Write down the row of coefficients in the expansion of each of the following.

(a) $(x + y)^8$

(b) $(x + y)^9$

6 The row of coefficients in the expansion of $(x + y)^n$ is $1, a, 55, b, 330, c, 462, d, 165, e, f, 1$.

(a) Find the values of a, b, c, d, e, f and n.

(b) Explain clearly how you obtained each value.

7 **(a)** Find the first three terms, in ascending powers of y, in the expansion of $(1 + 2y)^8$.

(b) By substituting an appropriate value of y, find an approximation to 1.02^8, correct to 3 significant figures.

8 **(a)** Expand $(3 + 2y)^4$.

(b) Hence, deduce **all** the terms in the expansion of $(3 - 2y)^4$.

(c) Simplify $(3 + 2y)^4 - (3 - 2y)^4$.

9 **(a)** Expand $(1 + 2x)^9$ up to and including the term in x^4.

(b) Find, in ascending powers of x, the first four terms in the expansion of $(2 + x)(1 + 2x)^9$.

(c) Find the term in x^3 in the expansion of $(2 + x)^2(1 + 2x)^9$.

10 Find the value of p for which the coefficient of x^3 in the expansion of $(3 - px)^5 + (x + 2)^6$, where p is an integer, is 880.

11 In the expansion of $(3 + kx)^9$, the coefficients of x^3 and x^4 are equal. Find the value of k.

Workbook pages for Section 12.2 on
Expansion of $(a+b)^n$ using Binomial Theorem
are not included in this submission.

Chapter Opener Revisit

Let's take a look at the question in the Chapter Opener of the Student's Book:

When you save money in a bank, you are paid a yearly interest. Suppose the bank offers you an annual rate of 1% compound interest and you put $\$500$ into a savings account for a total of 10 years. The exact amount in your account after 10 years is $\$500 \times 1.01^{10}$. How would you get an approximate answer without the use of a calculator?

Fun with Maths!

You have learnt two methods of expanding $(a + b)^n$ by applying the Pascal's Triangle, and the Binomial Theorem. You will now learn another interesting method for binomial expansion.

Here are the steps for expanding $(a + b)^n$:

Step 1: The first 1st term is a^n.

Step 2: The coefficient of the second 2nd term is n and its variable component is $a^{n-1}b$.

Step 3: For each subsequent term, its coefficient is can be found using the formula:

$$\frac{\text{Coefficient of the previous term} \times \text{Power of } a \text{ of the previous term}}{\text{Position of the previous term in the expansion}}$$

Its variable component can be derived from the decreasing powers of a and the increasing powers of b.

Step 4: The last term is b^n.

To expand $(a + b)^6$, note that there are 7 terms in the expansion of $(a + b)^6$.

<table>
<tr><td>Step 1</td><td>The 1st term is a^6.</td></tr>
<tr><td>Step 2</td><td>The coefficient of the 2nd term is 6. The variable component is a^5b.
So, the 2nd term is $6a^5b$.</td></tr>
<tr><td>Step 3</td><td>To find the 3rd term, use the 2nd term and the formula to work out its coefficient, $\frac{6 \times 5}{2} = 15$.
<table><tr><td>$6a^5b$</td><td>$\frac{6 \times 5}{2} = 15$</td></tr><tr><td>Term 2</td><td>Coefficient of term is 3</td></tr></table>
The variable component is a^4b^2. So, the 3rd term is $15a^4b^2$.</td></tr>
<tr><td>Step 4</td><td>Now using the 3rd term, the coefficient of the 4th term is $\frac{15 \times 4}{3} = 20$.
<table><tr><td>$15a^4b^2$</td><td>$\frac{15 \times 4}{3} = 20$</td></tr><tr><td>Term 3</td><td>Coefficient of term is 4</td></tr></table>
The variable is a^3b^3. So, the 4th term is $20a^3b^3$.</td></tr>
<tr><td>Step 5</td><td>The 5th and 6th terms can be worked out in a similar manner.
Since there are seven terms in this expansion, the 4th term is the middle term.
By symmetry, the 3rd and 5th terms have the same coefficient. Similarly, the 2nd and 6th terms have the same coefficient.
Therefore, the 5th term is $15a^2b^4$ and the 6th term is $6ab^5$.</td></tr>
<tr><td>Step 6</td><td>The 7th term is b^6.</td></tr>
</table>

Putting all the terms together, we have

$$(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

You may now apply the new method in binomial expansion.

1 Expand

(a) $(a + b)^{10}$ **(b)** $(3 - u)^8$ **(c)** $(x + 2y)^5$ **(d)** $(t^2 + 2)^7$ **(e)** $(p - \frac{2}{p^2})^6$

***2** Explain how the formula in Step 3 above is related to $\binom{n}{r}$.

CHAPTERS

12–14 Revision Exercise

1 Expand

(a) $(2 - a)^7$ as a sum of terms in ascending powers of a.

(b) $(2p - 3)^5$ as a sum of terms in descending powers of p.

2 Find the term which is independent of p in the expansion of $(p^2 - \frac{1}{2p^6})^{16}$.

3 **(a)** Find the first four terms in the expansion of $(2 + x^2)^6$ in ascending powers of x.

(b) The coefficient of x^4 in the expansion of $(1 + ax^2)(2 + x^2)^6$ is 48.
Find the value of a.

4 The first four terms in the expansion of $(2x - \frac{1}{4})^9$ are $512x^9 - 576x^8 + px^7 + qx^6 + \ldots$.

(a) Find the values of p and q.

(b) Calculate the coefficient of x^6 in the expansion of $(4x + 1)(2x - \frac{1}{4})^9$.

5 **(a)** Find, in ascending powers of y, the first four terms in the expansion of $(1 + y)^7$.

(b) The sum of the second and fourth terms in the expansion is twice the third term. Find the value of y.

6 **(a)** Find the first four terms in the expansion of $(y + \frac{1}{200y^2})^{10}$ in descending powers of y.

(b) Hence, find an approximation of $(1.005)^{10}$, correct to 4 significant figures.

(c) Explain why the powers of y in the expansion is never zero.